IMPORTANT FORMULAES

CLASS – 12

1. Relations & Functions

Definition/Theorems

- Empty relation holds a specific relation R in X as: **$R = \phi \subset X \times X$**.
- A Symmetric relation R in X satisfies a certain relation as: **(a, b) ∈ R implies (b, a) ∈ R**.
- A Reflexive relation R in X can be given as: **(a, a) ∈ R; for all ∀ a ∈ X**.
- A Transitive relation R in X can be given as: **(a, b) ∈ R and (b, c) ∈ R, thereby, implying (a, c) ∈ R**.
- A Universal relation is the relation R in X can be given by R = X × X.
- Equivalence relation R in X is a relation that shows all the reflexive, symmetric and transitive relations.

Properties

- A function f: X → Y is one-one/injective; if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2\ \forall\ x_1, x_2 \in X$.
- A function f: X → Y is onto/surjective; if given any y ∈ Y, ∃ x ∈ X such that f(x) = y.
- A function f: X → Y is one-one and onto or bijective; if f follows both the one-one and onto properties.
- A function f: X → Y is invertible if ∃ g: Y → X such that gof = I_X and fog = I_Y. This can happen only if f is one-one and onto.
- A binary operation ∗∗ performed on a set A is a function ∗∗ from A × A to A.
- An element e ∈ X possess the identity element for binary operation ∗∗ : X × X → X, if a ∗∗ e = a = e ∗∗ a; ∀ a ∈ X.
- An element a ∈ X shows the invertible property for binary operation ∗∗ : X × X → X, if there exists b ∈ X such that a ∗∗ b = e = b ∗∗ a where e is said to be the identity for the binary operation ∗∗. The element b is called the inverse of a and is denoted by a^{-1}.
- An operation ∗∗ on X is said to be commutative if a ∗∗ b = b ∗∗ a; ∀ a, b in X.
- An operation ∗∗ on X is said to associative if (a ∗∗ b) ∗∗ c = a ∗∗ (b ∗∗ c); ∀ a, b, c in X.

2. Inverse Trigonometric Functions

Properties/Theorems

The domain and range of inverse trigonometric functions are given below:

Functions	**Domain**	**Range**
$y = \sin^{-1} x$	[–1, 1]	$[-\pi 2,\pi 2][-\pi 2,\pi 2]$
$y = \cos^{-1} x$	[–1, 1]	$[0,\pi][0,\pi]$
$y = \text{cosec}^{-1} x$	R – (–1, 1)	$[-\pi 2,\pi 2][-\pi 2,\pi 2]$ – {0}
$y = \sec^{-1} x$	R – (–1, 1)	$[0,\pi][0,\pi]$ – $\{\pi 2\pi 2\}$
$y = \tan^{-1} x$	R	$(-\pi 2,\pi 2)(-\pi 2,\pi 2)$
$y = \cot^{-1} x$	R	$(0,\pi)(0,\pi)$

Formulas

- $y=\sin^{-1}x \Rightarrow x=\sin y$
- $x=\sin y \Rightarrow y=\sin^{-1}x$
- $\sin^{-1}1/x=\text{cosec}^{-1}x$
- $\cos^{-1}1/x=\sec^{-1}x$
- $\tan^{-1}1/x=\cot^{-1}x$
- $\cos^{-1}(-x)=\pi-\cos^{-1}x$
- $\cot^{-1}(-x)=\pi-\cot^{-1}x$
- $\sec^{-1}(-x)=\pi-\sec^{-1}x$
- $\sin^{-1}(-x)=-\sin^{-1}x$
- $\tan^{-1}(-x)=-\tan^{-1}x$
- $\text{cosec}^{-1}(-x)=-\text{cosec}^{-1}x$
- $\tan^{-1}x+\cot^{-1}x=\pi/2$
- $\sin^{-1}x+\cos^{-1}x=\pi/2$
- $\text{cosec}^{-1}x+\sec^{-1}x=\pi/2$
- $\tan^{-1}x+\tan^{-1}y=\tan^{-1}x+y/1-xy$
- $2\tan^{-1}x=\sin^{-1}2x/1+x^2=\cos^{-1}1-x^2/1+x^2$
- $2\tan^{-1}x=\tan^{-1}2x/1-x^2$
- $\tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}(x+y/1-xy)$; $xy > 1$; $x, y > 0$

3. Matrices

Definition/Theorems

- A matrix is said to have an ordered rectangular array of functions or numbers. A matrix of order m × n consists of m rows and n columns.
- An m × n matrix will be known as a square matrix when m = n.
- $A = [a_{ij}]_{m \times m}$ will be known as diagonal matrix if $a_{ij} = 0$, when $i \neq j$.
- $A = [a_{ij}]_{n \times n}$ is a scalar matrix if $a_{ij} = 0$, when $i \neq j$, $a_{ij} = k$, (where k is some constant); and i = j.
- $A = [a_{ij}]_{n \times n}$ is an identity matrix, if $a_{ij} = 1$, when i = j and $a_{ij} = 0$, when $i \neq j$.
- A zero matrix will contain all its element as zero.
- $A = [a_{ij}] = [b_{ij}] = B$ if and only if:
  1. (i) A and B are of the same order
  2. (ii) $a_{ij} = b_{ij}$ for all the certain values of i and j

Elementary Operations

- Some basic operations of matrices:
  1. (i) $kA = k[a_{ij}]_{m \times n} = [k(a_{ij})]_{m \times n}$
  2. (ii) – A = (– 1)A
  3. (iii) A – B = A + (– 1)B
  4. (iv) A + B = B + A
  5. (v) (A + B) + C = A + (B + C); where A, B and C all are of the same order
  6. (vi) k(A + B) = kA + kB; where A and B are of the same order; k is constant
  7. (vii) (k + l)A = kA + lA; where k and l are the constant
- If $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$, then
  $AB = C = [c_{ik}]_{m \times p}$; where c_{ik} = ∑nj=1aijbjk∑j=1naijbjk
  1. (i) A.(BC) = (AB).C
  2. (ii) A(B + C) = AB + AC
  3. (iii) (A + B)C = AC + BC
- If $A= [a_{ij}]_{m \times n}$, then A’ or AT = $[a_{ji}]_{n \times m}$
  1. (i) (A’)’ = A
  2. (ii) (kA)’ = kA’

3. (iii) (A + B)′ = A′ + B′
4. (iv) (AB)′ = B′A′

- Some elementary operations:
  1. (i) $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
  2. (ii) $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$
  3. (iii) $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$
- A is said to known as a symmetric matrix if A′ = A
- A is said to be the skew symmetric matrix if A′ = –A

4. Determinants

Definition/Theorems

- The determinant of a matrix $A = [a_{11}]_{1 \times 1}$ can be given as: $|a_{11}| = a_{11}$.
- For any square matrix A, the |A| will satisfy the following properties:
  1. |A′| = |A|, where A′ = transpose of A.
  2. If we interchange any two rows (or columns), then sign of determinant changes.
  3. If any two rows or any two columns are identical or proportional, then the value of the determinant is zero.
  4. If we multiply each element of a row or a column of a determinant by constant k, then the value of the determinant is multiplied by k.

Formulas

- Determinant of a matrix

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$ can be expanded as:

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

- Area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- Cofactor of aij of given by $A_{ij} = (-1)^{i+j} M_{ij}$
- If

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$ then,

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$ where A_{ij} is the cofactor of a_{ij}.

- $A^{-1}=(1/|A|)(\text{adj}A)$
- If $a_1x + b_1y + c_1z = d_1$ $a_2x + b_2y + c_2z = d_2$ $a_3x + b_3 y + c_3z = d_3$, then these equations can be written as A X = B, where:

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

- For a square matrix A in matrix equation AX = B
    1. (i) $|A| \neq 0$, there exists unique solution
    2. (ii) $|A| = 0$ and (adj A) B ≠ 0, then there exists no solution
    3. (iii) $|A| = 0$ and (adj A) B = 0, then the system may or may not be consistent.

5. Continuity and Differentiability

Definition/Properties

- A function is said to be continuous at a given point if the limit of that function at the point is equal to the value of the function at the same point.
- Properties related to the functions:
    - (i) $(f\pm g)(x)=f(x)\pm g(x)$ is continuous.
    - (ii) $(f.g)(x)=f(x).g(x)$ is continuous.
    - (iii) $f/g(x)=f(x)/g(x)$ (whenever $g(x)\neq 0$ $g(x)\neq 0$ is continuous.
- **Chain Rule:** If f = v o u, t = u (x) and if both dt/dx and dv/dx exists, then:

    - $df/dx=dv/dt.dt/dx$

- **Rolle's Theorem:** If f: [a, b] → R is continuous on [a, b] and differentiable on (a, b) where as f(a) = f(b), then there exists some c in (a, b) such that f ′(c) = 0.

- **Mean Value Theorem:** If f : [a, b] → R is continuous on [a, b] and differentiable on (a, b). Then there exists some c in (a, b) such that

$$f'(c)=f(b)-f(a)/b-a$$

Formulas

Given below are the standard derivatives:

Derivative	Formulas
$\frac{d}{dx}(\sin^{-1}x)$ d/dx(sin−1x)	$1/\sqrt{1-x^2}$
$\frac{d}{dx}(\cos^{-1}x)$ d/dx(cos−1x)	$-1/\sqrt{1-x^2}$
$\frac{d}{dx}(\tan^{-1}x)$ d/dx(tan−1x)	$1/1+x^2$
$\frac{d}{dx}(\cot^{-1}x)$ d/dx(cot−1x)	$-1/1+x^2$
$\frac{d}{dx}(\sec^{-1}x)$ d/dx(sec−1x)	$1/x\sqrt{1-x^2}$
$\frac{d}{dx}(\mathrm{cosec}^{-1}x)$ d/dx(cosec−1x)	$-1/x\sqrt{1-x^2}$
$\frac{d}{dx}(e^x)$ d/dx(ex)	e^x
$\frac{d}{dx}(\log x)$ d/dx(logx)	$1/x$

6. Integrals

Definition/Properties

- Integration is the inverse process of differentiation.
  Suppose, $\frac{d}{dx}F(x)=f(x)$; then we can write $/\int f(x)dx=F(x)+C$
- Properties of indefinite integrals:
  - (i) $\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$
  - (ii) For any real number k, $\int kf(x)dx=k\int f(x)dx$
  - (iii) $\int[k_1f_1(x)+k_2f_2(x)+\ldots+k_nf_n(x)]dx=k_1\int f_1(x)dx+k_2\int f_2(x)dx+\ldots+k_n\int f_n(x)dx$
- **First fundamental theorem of integral calculus:** Let the area function be defined as: $A(x)=\int_a^x f(x)dx$ for all $x\geq a$, where the function f is assumed to be continuous on [a, b]. Then A' (x) = f (x) for every x ∈ [a, b].
- **Second fundamental theorem of integral calculus:** Let f be the certain continuous function of x defined on the closed interval [a, b]; Furthermore, let's assume F another function as: $\frac{d}{dx}F(x)=f(x)$ for every x falling in the domain of f; then,

$$\int_a^b f(x)\,dx=[F(x)+C]_a^b=F(b)-F(a)$$

Formulas – Standard Integrals

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$. Particularly, $\int dx = x + C)$
2. $\int cos\, x\, dx = sin\, x + C$
3. $\int sin\, x\, dx = -cos\, x + C$
4. $\int sec^2 x\, dx = tan\, x + C$
5. $\int cosec^2 x\, dx = -cot\, x + C$
6. $\int sec\, x\, tan\, x\, dx = sec\, x + C$
7. $\int cosec\, x\, cot\, x\, dx = -cosec\, x + C$
8. $\int \frac{dx}{\sqrt{1-x^2}} = sin^{-1}x + C$
9. $\int \frac{dx}{\sqrt{1-x^2}} = -cos^{-1}x + C$
10. $\int \frac{dx}{1+x^2} = tan^{-1}x + C$
11. $\int \frac{dx}{1+x^2} = -cot^{-1}x + C$
12. $\int e^x dx = e^x + C$
13. $\int a^x dx = \frac{a^x}{log\, a} + C$
14. $\int \frac{dx}{x\sqrt{x^2-1}} = sec^{-1}x + C$
15. $\int \frac{dx}{x\sqrt{x^2-1}} = -cosec^{-1}x + C$
16. $\int \frac{1}{x}\, dx = log\, |x| + C$

Formulas – Partial Fractions

Partial Fraction	Formulas
$\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{x-a} + \frac{B}{x-b}, a \neq b$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-b)^2}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Formulas – Integration by Substitution

1. ∫tanxdx=log|secx|+C
2. ∫cotxdx=log|sinx|+C
3. ∫secxdx=log|secx+tanx|+C
4. ∫cosecxdx=log|cosecx−cotx|+C

Formulas – Integrals (Special Functions)

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} log \left|\frac{x-a}{x+a}\right| + C$
2. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} log \left|\frac{a+x}{a-x}\right| + C$
3. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} tan^{-1} \frac{x}{a} + C$
4. $\int \frac{dx}{\sqrt{x^2-a^2}} = log \left|x + \sqrt{x^2-a^2}\right| + C$
5. $\int \frac{dx}{\sqrt{x^2+a^2}} = log \left|x + \sqrt{x^2+a^2}\right| + C$
6. $\int \frac{dx}{\sqrt{x^2-a^2}} = sin^{-1} \frac{x}{a} + C$

Formulas – Integration by Parts

1. The integral of the product of two functions = first function × integral of the second function – integral of {differential coefficient of the first function × integral of the second function}
∫f1(x).f2(x)=f1(x)∫f2(x)dx−∫[ddxf1(x).∫f2(x)dx]dx
2. ∫ex[f(x)+f′(x)]dx=∫exf(x)dx+C

Formulas – Special Integrals

1. $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2-a^2}\right| + C$
2. $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2+a^2}\right| + C$
3. $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a}{2} sin^{-1} \frac{x}{a} + C$
4. $ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right]$

7. Application of Integrals

1. The area enclosed by the curve y = f (x) ; x-axis and the lines x = a and x = b (b > a) is given by the formula:
$$Area = \int_a^b y\, dx = \int_a^b f(x)\, dx$$
2. Area of the region bounded by the curve x = φ (y) as its y-axis and the lines y = c, y = d is given by the formula:
$$Area = \int_c^d x\, dy = \int_c^d \phi(y)\, dy$$
3. The area enclosed in between the two given curves y = f (x), y = g (x) and the lines x = a, x = b is given by the following formula:
$$Area = \int_a^b [f(x) - g(x)]\, dx,\ where,\ f(x) \geq g(x)\ in\ [a, b]$$
4. If f (x) ≥ g (x) in [a, c] and f (x) ≤ g (x) in [c, b], a < c < b, then:
$$Area = \int_a^c [f(x) - g(x)]\, dx, + \int_c^b [g(x) - f(x)]\, dx$$

8. Vector Algebra

Definition/Properties

1. Vector is a certain quantity that has both the magnitude and the direction. The position vector of a point P (x, y, z) is given by:

   OP−→−(=r⃗)=xi^+yj^+zk^

2. The scalar product of two given vectors a⃗ a→ and b⃗ b→ having angle θ between them is defined as:

   a⃗ .b⃗ =|a⃗ ||b⃗ |cosθ

3. The position vector of a point R dividing a line segment joining the points P and Q whose position vectors a⃗ a→ and b⃗ b→ are respectively, in the ratio m : n is given by:

- (i) internally: na⃗ +mb⃗ m+n
- (ii) externally: na⃗ −mb⃗ m−n

Formulas

If two vectors a⃗ a→ and b⃗ b→ are given in its component forms as a^=a1i^+a2j^+a3k^ and b^=b1i^+b2j^+b3k^ and λ as the scalar part; then:

(i) $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$;

(ii) $\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$;

(iii) $\vec{a} \, . \, \vec{b} = (a_1 b_1) + (a_2 b_2) + (a_3 b_3)$

(iv) and $\vec{a} \times \vec{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$.

9. Three dimensional Geometry

Definition/Properties

- Direction cosines of a line are the cosines of the angle made by a particular line with the positive directions on coordinate axes.
- Skew lines are lines in space which are neither parallel nor intersecting. These lines lie in separate planes.
- If l, m and n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.

Formulas

1. The Direction cosines of a line joining two points P (x_1 , y_1 , z_1) and Q (x_2 , y_2 , z_2) are $\frac{x_2-x_1}{PQ}$, $\frac{y_2-y_1}{PQ}$, $\frac{z_2-z_1}{PQ}$ where
PQ=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

2. Equation of a line through a point (x_1 , y_1 , z_1) and having direction cosines l, m, n is: $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$
3. The vector equation of a line which passes through two points whose position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$
4. The shortest distance between $\vec{r}=\overrightarrow{a_1}+\lambda\,\overrightarrow{b_1}$ and $\vec{r}=\overrightarrow{a_2}+\mu\,\overrightarrow{b_2}$ is:
$$\left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$$

5. The distance between parallel lines $\vec{r}=\overrightarrow{a_1}+\lambda\,\vec{b}$ and $\vec{r}=\overrightarrow{a_2}+\mu\,\vec{b}$ is
$$\left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}|}\right|$$

6. The equation of a plane through a point whose position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $(\vec{r}-\vec{a})\,.\,\vec{N}=0$
7. Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point (x_1 , y_1 , z_1) is A (x – x_1) + B (y – y_1) + C (z – z_1) = 0
8. The equation of a plane passing through three non-collinear points (x_1 , y_1 , z_1); (x_2 , y_2 , z_2) and (x_3 , y_3 , z_3) is:
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

9. The two lines $\vec{r}=\overrightarrow{a_1}+\lambda\,\overrightarrow{b_1}$ and $\vec{r}=\overrightarrow{a_2}+\mu\,\overrightarrow{b_2}$ are coplanar if:
$$(\overrightarrow{a_2}-\overrightarrow{a_1})\,.\,(\overrightarrow{b_1}\times\overrightarrow{b_2})=0$$

10. The angle φ between the line $\vec{r}=\vec{a}+\lambda\,\vec{b}$ and the plane $\vec{r}\,.\,\hat{n}=d$ is given by:
$$sin\,\phi=\left|\frac{\vec{b}\,.\,\hat{n}}{|\vec{b}||\hat{n}|}\right|$$

11. The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by:
$$cos\,\theta=\left|\frac{A_1\,A_2+B_1\,B_2+C_1\,C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\,\sqrt{A_2^2+B_2^2+C_2^2}}\right|$$

12. The distance of a point whose position vector is $\vec{a}$ from the plane $\vec{r}\,.\,\hat{n}=d$ is given by: $|d-\vec{a}\,.\,\hat{n}|$
13. The distance from a point (x_1 , y_1 , z_1) to the plane Ax + By + Cz + D = 0:
$$\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$$

10. Probability

Definition/Properties

1. The conditional probability of an event E holds the value of the occurrence of the event F as:

$$P(E \mid F) = \frac{E \cap F}{P(F)}\ ,\ P(F) \neq 0$$

2. **Total Probability:** Let E_1 , E_2 , ... , E_n be the partition of a sample space and A be any event; then,

$$P(A) = P(E_1)\ P(A|E_1) + P(E_2)\ P(A|E_2) + \ldots + P(E_n)\ .\ P(A|E_n)$$

3. **Bayes Theorem:** If E_1 , E_2 , ... , E_n are events contituting in a sample space S; then,

$$P(E_i \mid A) = \frac{P(E_i)\ P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\ P(A|E_j)}$$

4. Var (X) = E (X^2) – $[E(X)]^2$