CBSE Class 12 Maths Question Paper Solution 2019

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$ $\frac{1}{2}$

$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$ $\frac{1}{2}$

2. $(fof)(x) = f(x+1) = x+2$ $\frac{1}{2}$

$\frac{d}{dx}(fof)(x) = 1$ $\frac{1}{2}$

3. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= <0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k})+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. As $a, b \in R \Rightarrow ab \in R \Rightarrow ab+1 \in R \Rightarrow a*b \in R \Rightarrow *$ is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general $(a*b)*c \neq a*(b*c) \Rightarrow *$ is not associative. 1

6. $2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ 1

7. Put tan x = t ⇒ $\sec^2 x\ dx = dt$ $\frac{1}{2}$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2+4}} = \log|t + \sqrt{t^2+4}| + C$$ 1

$$= \log|\tan x + \sqrt{\tan^2 x + 4}| + C$$ $\frac{1}{2}$

8. Let $I = \int \sqrt{1 - \sin 2x}\ dx$

$= \int (\sin x - \cos x) dx$ as $\sin x > \cos x$ when $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ 1

$= -\cos x - \sin x + C$ 1

OR

$$I = \int \sin^{-1}(2x).1\, dx$$

$$= x.\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}} dx$$ 1

$$= x.\sin^{-1}(2x) + \frac{1}{4}\int \frac{-8x}{\sqrt{1-4x^2}} dx = x \sin^{-1}(2x) + \frac{1}{2}.\sqrt{1-4x^2} + C$$ 1

9. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$$\frac{e^{2x}.(y'' - 2y') - (y' - 2y).2x^{2x}}{(e^{2x})^2} = 0$$ 1

$$\Rightarrow y'' - 4y' + 4y = 0 \text{ or } \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$ $\frac{1}{2}$

10. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow 1 + |\vec{a} - \vec{b}|^2 = 2(1+1)$

$\Rightarrow |\hat{a}-\hat{b}|^2 = 3 \Rightarrow |\hat{a}-\hat{b}| = \sqrt{3}$ 1

OR

$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$ 1

$= -30$ 1

11. A = {2, 4, 6}, B = {1, 2, 3}, $A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2},\ P(B) = \frac{1}{2},\ P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

12. Let X: getting an odd number

$p = \frac{1}{2},\ q = \frac{1}{2},\ n = 6$ $\frac{1}{2}$

(i) $P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \le 5) = 1 - P(X = 6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow k = \frac{1}{6}$ 1

13. Clearly $a \le a\ \forall\ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow$ a $\leq$ b and b $\leq$ c $\Rightarrow$ a $\leq$ c $\Rightarrow$ (a, c) $\in$ R

$\Rightarrow$ R is transitive. $1\frac{1}{2}$

For non-symmetric:

Let a = 1, b = 2. As $1 \leq 2 \Rightarrow (1, 2) \in R$ but $2 \not\leq 1 \Rightarrow (2,1) \notin R$

$\Rightarrow$ R is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$$

$$\Rightarrow x_1 = x_2 \quad \text{as} \quad x_1 + x_2 + 1 \neq 0 \qquad (\because x_1, x_2 \in N)$$ $1\frac{1}{2}$

$$\Rightarrow f \text{ is one-one.}$$

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$$\therefore f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2} \quad \text{or} \quad f^{-1}(x) = \frac{\sqrt{4x-3}-1}{2}$$ 1

14. $\tan^{-1}\left(\frac{4x+6x}{1-(4x)(6x)}\right) = \frac{\pi}{4}$ 1

$$\Rightarrow \frac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$$ $1\frac{1}{2}$

$$\Rightarrow x = \frac{1}{12} \text{ or } -\frac{1}{2}$$ 1

as $x = -\frac{1}{2}$ does not satisfy the given equation, so $x = \frac{1}{12}$ $\frac{1}{2}$

15. LHS $= \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 2

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding along C_3,

$$= (a-1)^2.(a-1) = (a-1)^3 = \text{RHS}.$$ 1

16. $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right) = 2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right) = \frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$$ 1

$$\Rightarrow (x+y) = (x-y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx} - \frac{dv}{dx} = 0 \qquad \text{...(1)} \qquad 1$$

Now, log u = y.logx

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} \Rightarrow \frac{du}{dx} = x^y \left(\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right) \qquad \text{...(2)} \qquad 1$$

Again, log v = x · log y

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1 \Rightarrow \frac{dv}{dx} = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) \qquad \text{...(3)} \qquad 1$$

From (1), (2) and (3)

$$x^y \left(\frac{y}{x} + \log x \frac{dy}{dx} \right) - y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = 0 \qquad \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x \cdot \log y - x^{y-1} \cdot y}{x^y \cdot \log x - y^{x-1} \cdot x} \qquad \frac{1}{2}$$

17. $y = (\sin^{-1} x)^2$

$$\Rightarrow y' = 2 \cdot \sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}} \qquad 1$$

$$\Rightarrow \sqrt{1-x^2} \cdot y' = 2 \sin^{-1} x$$

$$\Rightarrow \sqrt{1-x^2} \cdot y'' + y' \cdot \frac{1}{2\sqrt{1-x^2}} (-2x) = \frac{2}{\sqrt{1-x^2}} \qquad 2$$

$$\Rightarrow (1-x^2) \cdot y'' - xy' = 2 \text{ or } (1-x^2) \cdot \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0. \qquad 1$$

18. Let the point of contact be $P(x_1, y_1)$

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} \qquad \text{(slope of tangent)}$$

$$\Rightarrow m_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1 - 2}} \qquad 1$$

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \frac{41}{48}$ 1

when $x_1 = \frac{41}{48}, y_1 = \sqrt{\frac{41}{16} - 2} = \frac{3}{4}$ $\therefore P\left(\frac{41}{48}, \frac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

19. $I = \int \frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{x^2+3x-18}\,dx$ 1

$= \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx$ 1

$= \frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$ 1 + 1

20. Let $I = \int_0^a f(a-x)\,dx$

Put $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$I = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$ $\frac{1}{2}$

II part.

$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}dx$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{(\pi - x)\cdot \sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1+\cos^2 x}\,dx \qquad 1\frac{1}{2}$$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow \ I = -\frac{\pi}{2}\cdot\int_1^{-1}\frac{dt}{1+t^2} = \frac{\pi}{2}\times 2\times\int_0^1\frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4} \qquad 1\frac{1}{2}$$

21. Writing $\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$

$$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x} \qquad 1$$

$$\Rightarrow \log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log c \qquad 1$$

$$\Rightarrow \ v+\sqrt{1+v^2} = cx \ \Rightarrow \ y+\sqrt{x^2+y^2} = cx^2$$

when x = 1, y = 0 ⇒ c = 1 $\frac{1}{2}$

$$\therefore \ y+\sqrt{x^2+y^2} = x^2 \qquad \frac{1}{2}$$

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{4x^2}{1+x^2}$ $\frac{1}{2}$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = 1+x^2 \qquad 1$$

Solution is given by,

$$y \cdot (1+x^2) = \int \frac{4x^2}{1+x^2} \cdot (1+x^2)\,dx = \int 4x^2\,dx \qquad 1$$

$$\Rightarrow \; y \cdot (1+x^2) = \frac{4x^3}{3} + c \qquad \frac{1}{2}$$

when $x = 0,\ y = 0 \Rightarrow c = 0$ $\quad \frac{1}{2}$

$$y \cdot (1+x^2) = \frac{4x^3}{3} \quad \text{or} \quad y = \frac{4x^3}{3(1+x^2)} \qquad \frac{1}{2}$$

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ $\quad 1$

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ $\quad 1$

Let required angle be θ.

$$\text{Then } \cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \frac{-2-32-2}{\sqrt{18}\,\sqrt{72}} = -1 \qquad 1$$

$\Rightarrow \theta = 180°$ or π $\quad \frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\quad \frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ $\quad 1$

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7 \qquad 1$$

So, lines are

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{x-3}{2} \quad \text{and} \quad \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5} \qquad \frac{1}{2}$$

Consider $\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ 2

$$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as $AX = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$ 1

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ 1

$\Rightarrow x = 3, y = 1, z = 2$ $\frac{1}{2}$

OR

$A = I.A$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A$$

4

$R_3 \to 5R_3$

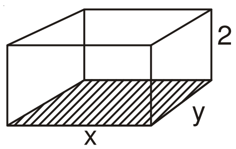
$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{6}{5}R_3, R_2 \to R_2 + \frac{2}{5}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

25.



$V = 2xy \Rightarrow 2xy = 8$ (given) 1

$\Rightarrow y = \dfrac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ 1

$= 280 + 180x + \dfrac{720}{x}$ 1

$\dfrac{dC}{dx} = 180 - \dfrac{720}{x^2}$ 1

$\dfrac{dC}{dx} = 0 \Rightarrow x = 2\text{m}$ $\frac{1}{2}$

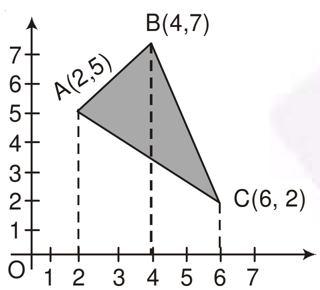
$\dfrac{d^2C}{dx^2} = \dfrac{1440}{x^3} = 180 > 0$ at $x = 2$ $\frac{1}{2}$

$\Rightarrow$ C is minimum at $x = 2\text{m}$. $\frac{1}{2}$

Minimum cost $= 280 + 180(2) + \dfrac{720}{2} =$ ₹ 1,000 $\frac{1}{2}$

26.

Correct Figure 1



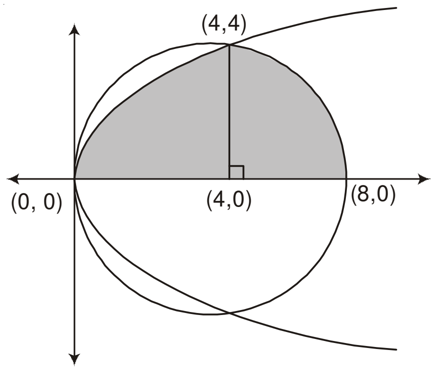
$$\left.\begin{array}{l} \text{Equation of AB}: y = x + 3 \\ \text{Equation of BC}: y = \dfrac{-5x}{2} + 17 \\ \text{Equation of AC}: y = \dfrac{-3x}{4} + \dfrac{13}{2} \end{array}\right\}$$ $1\frac{1}{2}$

Required Area $= \displaystyle\int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2} + 17\right) dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2}\right) dx$ $1\frac{1}{2}$

$= \left[\dfrac{(x+3)^2}{2}\right]_2^4 + \left[\dfrac{-5x^2}{4} + 17x\right]_4^6 - \left[\dfrac{-3x^2}{8} + \dfrac{13x}{2}\right]_2^6$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

Required Area $= \int_0^4 2\sqrt{x}\, dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\, dx$ $1\frac{1}{2}$

$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$ $1\frac{1}{2}$

$= \left(4\pi + \frac{32}{3}\right)$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\dfrac{|-2+1+4|}{\sqrt{(-1)^2 + 1^2 + 1^2}} = \sqrt{3}$ 1

28.

$$\left.\begin{array}{l} \text{Let } E_1 : \text{item is produced by A} \\ \quad E_2 : \text{item is produced by B} \\ \quad E_3 : \text{item is produced by C} \\ \quad A : \text{defective item is found.} \end{array}\right\}$$ 1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$ 1

$$P(A/E_1) = \frac{1}{100}, P(A/E_2) = \frac{5}{100}, P(A|E_3) = \frac{7}{100}$$ 1

$$P(E_1|A) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$ 2

$$= \frac{5}{34}$$ 1

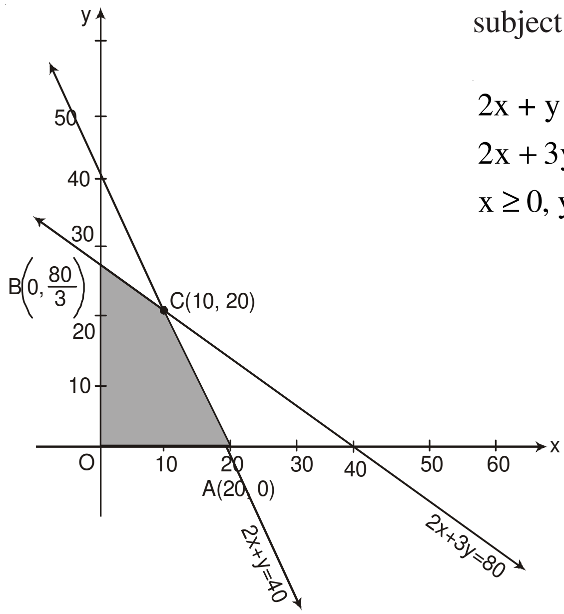
29. Let number of items produced of model A be x and that of model B be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\}$$ 2



Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10$, $y = 20$. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.