CLASS – 12

CHAPTER -9 Differential Equations

Differential Equation

An equation involving independent variable, dependent variable, derivatives of dependent variable with respect to independent variable and constant is called a differential equation.
e.g.

$$x\frac{dy}{dx}+xy\frac{d^2y}{dx^2}+4=0,\ \frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0.$$

Ordinary Differential Equation: An equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.
e.g.

$$\frac{dy}{dx}+\frac{d^2y}{dx^2}-2=0.$$

From any given relationship between the dependent and independent variables, a differential equation can be formed by differentiating it with respect to the independent variable and eliminating arbitrary constants involved.

Order of a Differential Equation: Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.
Note: Order of the differential equation, cannot be more than the number of arbitrary constants in the equation.

Degree of a Differential Equation: The highest exponent of the highest order derivative is called the degree of a differential equation provided exponent of each derivative and the unknown variable appearing in the differential equation is a non-negative integer.

Note:-
(i) Order and degree (if defined) of a differential equation are always positive integers.
(ii) The differential equation is a polynomial equation in derivatives.
(iii) If the given differential equation is not a polynomial equation in its derivatives, then its degree is not defined.

Formation of a Differential Equation: To form a differential equation from a given relation, we use the following steps:
Step I: Write the given equation and see the number of arbitrary constants it has.
Step II: Differentiate the given equation with respect to the dependent variable n times, where n is the number of arbitrary constants in the given equation.
Step III: Eliminate all arbitrary constants from the equations formed after differentiating in step (II) and the given equation.
Step IV: The equation obtained without the arbitrary constants is the required differential equation.

Solution of the Differential Equation
A function of the form $y = \Phi(x) + C$, which satisfies given differential equation, is called the solution of the differential equation.
General solution: The solution which contains as many arbitrary constants as the order of the differential equation, is called the general solution of the differential equation, i.e. if the solution of a differential equation of order n contains n arbitrary constants, then it is the general solution.

Particular solution: A solution obtained by giving particular values to arbitrary constants in the general solution of a differential equation, is called the particular solution.

Methods of Solving First Order and First Degree Differential Equation

Variable separable form: Suppose a differential equation is $dy/dx = F(x, y)$.
Here, we separate the variables and then integrate both sides to get the general solution, i.e. above equation may be written as $dy/dx = h(x) . k(y)$
Then, by separating the variables, we get $dy/k(y) = h(x)\, dx$.
Now, integrate above equation and get the general solution as $K(y) = H(x) + C$
Here, $K(y)$ and $H(x)$ are the anti-derivatives of $1K(y)$ and $h(x)$, respectively and C is the arbitrary constant.

Homogeneous differential equation: A differential equation $dy/dx=f(x,y)/g(x,y)$ is said to be homogeneous, if f(x, y) and g(x, y) are homogeneous functions of same degree, i.e. it may be written as

$$\frac{dy}{dx}=\frac{x^n f\left(\frac{y}{x}\right)}{x^n g\left(\frac{y}{x}\right)}=\frac{f(y/x)}{g(y/x)}=F(y/x) \quad \text{.......(i)}$$

To check that given differential equation is homogeneous or not, we write differential equation as $dy/dx = F(x, y)$ or $dx/dy = F(x, y)$ and replace x by λx, y by λy to write $F(x, y) = \lambda F(x, y)$.
Here, if power of λ is zero, then differential equation is homogeneous, otherwise not.

Solution of homogeneous differential equation: To solve homogeneous differential equation, we put
$y = vx$
$\Rightarrow dy/dx = v + x\, dv/dx$
in Eq. (i) to reduce it into variable separable form. Then, solve it and lastly put $v = y/x$ to get required solution.

Note: If the homogeneous differential equation is in the form of $dy/dx = F(x, y)$, where F(x, y) is homogeneous function of degree zero, then we make substitution $x/y = v$, i.e. $x = vy$ and we proceed further to find the general solution as mentioned above.

Linear differential equation: General form of linear differential equation is
$dy/dx + Py = Q$ …(i)
where, P and Q are functions of x or constants.
or $dx/dy + P'x = Q'$ …(ii)
where, P' and Q' are functions of y or constants.
Then, solution of Eq. (i) is given by the equation
$y \times IF = \int(Q \times IF)\, dx + C$
where, IF = Integrating factor and $IF = e^{\int P dx}$
Also, solution of Eq. (ii) is given by the equation
$x \times IF = \int (Q' \times IF)\, dy + C$
where, IF = Integrating factor and $IF = e^{\int P' dy}$