CLASS – 12

CHAPTER -3 Matrices

Matrices

Matrix: A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

Order of a Matrix: If a matrix has m rows and n columns, then its order is written as m × n. If a matrix has order m × n, then it has mn elements.

In general, a$_{m\times n}$ matrix has the following rectangular array:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n} \quad \text{or} \quad A=[a_{ij}]_{m\times n},\ 1\le i\le m, i\le j\le n; i,j\in N$$

Note: We shall consider only those matrices, whose elements are real numbers or functions taking real values.

Types of Matrices

Column Matrix: A matrix which has only one column, is called a column matrix.
e.g. [10−5]
In general, A = $[a_{ij}]_{m\times 1}$ is a column matrix of order m × 1.

Row Matrix: A matrix which has only one row, is called a row matrix,
e.g. [159]
In general, A = $[a_{ij}]_{1\times n}$ is a row matrix of order 1 x n

Square Matrix: A matrix which has equal number of rows and columns, is called a square matrix
e.g. [35−12]
In general, A = $[a_{ij}]$m x m is a square matrix of order m.
Note: If A = $[a_{ij}]$ is a square matrix of order n, then elements a_{11}, a_{22}, a_{33},…, a_{nn} is said to constitute the diagonal of the matrix A.

Diagonal Matrix: A square matrix whose all the elements except the diagonal elements are zeroes, is called a diagonal matrix,
e.g. [3000−3000−8]
In general, A = $[a_{ij}]_{m\times m}$ is a diagonal matrix, if a_{ij} = 0, when i ≠ j.

Scalar Matrix: A diagonal matrix whose all diagonal elements are same (non-zero), is called a scalar matrix,
e.g. ⎡⎣⎢200020002⎤⎦⎥
In general, A = $[a_{ij}]_{n×n}$ is a scalar matrix, if a_{ij} = 0, when i ≠ j, a_{ij} = k (constant), when i = j.
Note: A scalar matrix is a diagonal matrix but a diagonal matrix may or may not be a scalar matrix.

Unit or Identity Matrix: A diagonal matrix in which all diagonal elements are '1' and all non-diagonal elements are zero, is called an identity matrix. It is denoted by I.
e.g. ⎡⎣⎢100010001⎤⎦⎥
In general, A = $[a_{ij}]_{n×n}$ is an identity matrix, if a_{ij} = 1, when i = j and a_{ij} = 0, when i ≠ j.

Zero or Null Matrix: A matrix is said to be a zero or null matrix, if its all elements are zer0
e.g. [0000]

Equality of Matrices: Two matrices A and B are said to be equal, if
(i) order of A and B are same.
(ii) corresponding elements of A and B are same i.e. a_{ij} = b_{ij}, ∀ i and j.
e.g. [2013] and [2013] are equal matrices, but [3021] and [2031] are not equal matrices.

Operations on Matrices

Between two or more than two matrices, the following operations are defined below:
Addition and Subtraction of Matrices: Addition and subtraction of two matrices are defined in an order of both the matrices are same.
Addition of Matrix
If A = $[a_{ij}]_{m×n}$ and B = $[y_{ij}]_{m×n}$, then A + B = $[a_{ij} + b_{ij}]_{m×n}$, 1 ≤ i ≤ m, 1 ≤ j ≤ n
Subtraction of Matrix
If A = $[a_{ij}]_{m×n}$ and B = $[b_{ij}]_{m×n}$, then A – B = $[a_{ij} – b_{ij}]_{m×n}$, 1 ≤ i ≤ m, 1 ≤ j ≤ n

Properties of Addition of Matrices

(a) Commutative If A = $[a_{ij}]$ and B = $[b_{ij}]$ are matrices of the same order say m x n then A + B = B + A,
(b) Associative for any three matrices A = $[a_{ij}]$, B = $[b_{ij}]$, C = $[c_{ij}]$ of the same order say m x n, A + (B + C) = (A + B) + C.

(c) Existence of additive identity Let A = [aij] be amxn matrix and O be amxn zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.
(d) Existence of additive inverse Let $A = [a_{ij}]_{m\times n}$ be any matrix, then we have another matrix as $-A = [-a_{ij}]_{m\times n}$ such that A + (-A) = (-A + A) = O. So, matrix (-A) is called additive inverse of A or negative of A.

Note

(i) If A and B are not of the same order, then A + B is not defined.
(ii) Addition of matrices is an example of a binary operation on the set of matrices of the same order.

Multiplication of a matrix by scalar number: Let $A = [a_{ij}]_{m\times n}$ be a matrix and k is scalar, then kA is another matrix obtained by multiplying each element of A by the scalar k, i.e. if $A = [a_{ij}]_{m\times n}$, then $kA = [ka_{ij}]_{m\times n}$.

e.g. $$k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}_{2\times 2}$$

Properties of Scalar Multiplication of a Matrix
Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same order say m × n, then
(a) k(A + B) = kA + kB, where k is a scalar.
(b) (k + l)A = kA + lA, where k and l are scalars.

Multiplication of Matrices: Let A and B be two matrices. Then, their product AB is defined, if the number of columns in matrix A is equal to the number of rows in matrix B.

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$, then product $AB = C = [c_{ik}]_{m\times p}$, where $c_{ik} = \sum_{j=1}^{n} a_{ij}b_{jk}$.

In other words, if $A = [aij]_{m\times n}$, $B = [b_{jk}]_{n\times n}$, then the ith row of A is $[a_{i1} \ a_{i2} \ \dots \ a_{in}]$ and the kth column of B is $\begin{bmatrix} b_{1k} \\ b_{2k} \\ \vdots \\ b_{nk} \end{bmatrix}$, then $C_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \dots + a_{in}b_{nk} = \sum_{j=1}^{n} a_{ij}b_{jk}$.

Properties of Multiplication of Matrices

(a) Non-commutativity Matrix multiplication is not commutative i.e. if AB and BA are both defined, then it is not necessary that AB ≠ BA.
(b) Associative law For three matrices A, B, and C, if multiplication is defined,

then A (BC) = (AB) C.
(c) Multiplicative identity For every square matrix A, there exists an identity matrix of the same order such that IA = AI = A.
Note: For Amxm, there is only one multiplicative identity I_m.
(d) Distributive law For three matrices A, B, and C,
A(B + C) = AB + AC
(A + B)C = AC + BC
whenever both sides of the equality are defined.

Note: If A and B are two non-zero matrices, then their product may be a zero matrix.
e.g. Suppose A = [00−12] and B = [3050], then AB = [0000].