CLASS – 12

CHAPTER -1 Relations and Functions

Relations

A relation can be mathematically defined as the linking or connection between two different objects or quantities.

Examples of relations:

- {(a, b) ∈ A × B: a is the brother of b},
- {(a, b) ∈ A × B: a is the sister of b},
- {(a, b) ∈ A × B: age of a is greater than the age of b},
- {(a, b) ∈ A × B: total marks obtained by a in the final examination is less than the total marks obtained by b in the final examination},
- {(a, b) ∈ A × B: a lives in the same locality as b}. However, abstracting from this, we define mathematically a relation R from A to B as an arbitrary subset of A*x*

Types of Relations

- Empty Relation
- Universal Relation
- Reflexive Relation
- Symmetric relation
- Transitive relation
- Equivalence relation

Empty Relation: A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., R = ϕ ⊂ A × A.

Universal Relation: A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., R = A × A.

Reflexive Relation: A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Symmetric relation R in X is a relation satisfying (a, b) ∈ R implies (b, a) ∈ R.

Transitive relation R in X is a relation satisfying (a, b) ∈ R and (b, c) ∈ R implies that (a, c) ∈ R.

Equivalence relation R in X is a relation which is reflexive, symmetric and transitive.

Functions

Functions are defined as a special kind of relations.

Types of Functions

1) One-one Function

A function f : X → Y is one-one (or injective) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in X$.

2) Onto Function

A function f : X → Y is onto (or surjective) if given any y ∈ Y, ∃ x ∈ X such that f(x) = y.

3) One-One and Onto Function

A function f : X → Y is one-one and onto (or bijective), if f is both one-one and onto.

Composition of functions

The composition of functions f : A → B and g : B → C is the function gof : A → C given by

$gof(x) = g(f(x)) \ \forall \ x \in A$.

Invertible Function

A function f : X → Y is invertible if ∃ g : Y → X such that gof = I_X and fog = I_Y.

Condition- A function f : X → Y is invertible if and only if f is one-one and onto.

Binary Operation

A binary operation can be defined as the set of operations such as addition, subtraction, division and multiplication that are usually carried out to an arbitrary set called 'X'. The operations that ensue, in order to obtain a result for the following pair of elements such a, b from X to another element of X is called as a binary operation.

A binary operation $*$ on a set A is a function $*$ from $A \times A$ to A.

Properties

- An element $e \in X$ is the identity element for binary operation $* : X \times X \to X$, if $a * e = a = e * a \; \forall \; a \in X$
- An element $a \in X$ is invertible for binary operation $* : X \times X \to X$, if there exists $b \in X$ such that $a * b = e = b * a$ where, e is the identity for the binary operation $*$. The element b is called inverse of a and is denoted by a^{-1} .
- An operation $*$ on X is commutative if $a * b = b * a \; \forall$ a, b in X.
- An operation $*$ on X is associative if $(a * b) * c = a * (b * c) \forall$ a, b, c in X.