CBSE Class 10 Maths Question Paper Solution 2010

QUESTION PAPER CODE 30/1/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

			Marks
1. Terminating	2. $x^2 - 6x + 4$	3. $d = 2a$	$1 \times 10 = 10m$
4. 1 : 9	5. 5cm	6. $\frac{1}{3}$	
7. $p = 3$	8. (3.5)	9. $48cm^2$	
10. $\frac{3}{26}$			

SECTION - B

11. $p(x) = x^3 - 4x^2 - 3x + 12$

$\sqrt{3}$ and $-\sqrt{3}$ are zeroes of $p(x)$ $\Rightarrow$ $(x^2 - 3)$ is a factor of $p(x)$ ½ m

$$\left(x^3 - 3x - 4x^2 + 12\right) \div \left(x^2 - 3\right) = x - 4$$ 1m

$\Rightarrow$ $x = 4$ is the third zero of $p(x)$ ½ m

12. For a pair of linear equations to have infinitely

many solutions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ½ m

$$\therefore \overset{(i)}{\frac{2}{k-1}} = \overset{(ii)}{\frac{3}{k+2}} = \overset{(iii)}{\frac{7}{3k}}$$

From (i) and (ii) getting $k = 7$ 1 m

$k = 7$ satisfies (ii) and (iii) and (i) and (iii) also ½ m

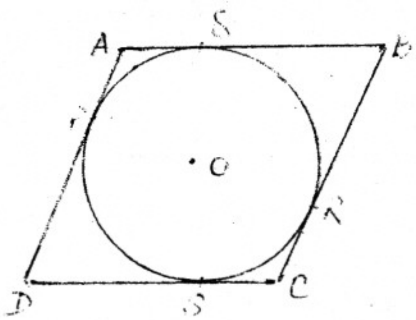
$\therefore \quad k = 7$

13. Here $a = 2,\ \ell = 29$ and $s_n = 155$ — 1 m

$$\therefore\ 155 = \frac{n}{2}\left[2+29\right] \Rightarrow n = 10$$

Also, $29 = 2 + (10-1)\,d \Rightarrow d = 3$

$\therefore$ Common difference = 3 — 1 m

14. 

As parallelogram ABCD circumscribes a circle with centre O

$\therefore$ AB + CD = BC + AD

[$\because$ AQ = AP, BQ = BR, CR = CS, PD = DS] — ½ m

As ABCD is a parallelogram $\Rightarrow$ AB = DC

and BC = AD

2AB = 2AD or AB = AD — 1 m

$\therefore$ ABCD is a rhombus (As AB = BC = CD = AD) — ½ m

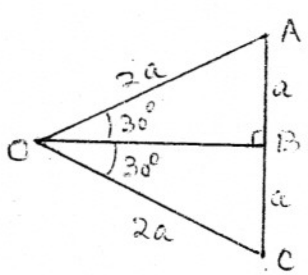
15. $\sec\left(90^\circ-\theta\right) = \operatorname{cosec}\theta,\ \tan\left(90^\circ-\theta\right) = \cot\theta,\ \cos 65^\circ = \sin 25^\circ$

and $\tan 63^\circ = \cot 27^\circ$ — 1 m

$\therefore$ Given expression becomes

$$\frac{\left(\operatorname{cosec}^2\theta-\cot^2\theta\right)+\left(\cos^2 25^\circ+\sin^2 25^\circ\right)}{3\tan 27^\circ\cot 27^\circ} = \frac{1+1}{3} = \frac{2}{3}$$ — 1 m

OR



correct Fig. — ½ m

Draw rt. Δ OBA, in which $\angle BOA = 30^\circ$

Take OA = 2a. Replicate Δ OCB on the other side of OB $\Rightarrow$ $\angle AOC = 60^\circ$ — ½ m

and OC = 2a

$\therefore$ Δ AOC is equilateral Δ and AB = a

$$\text{cosec } 30^\circ = \frac{OA}{AB} = \frac{2a}{a} = 2$$

$$\therefore \text{cosec } 30^\circ = 2$$ 1m

SECTION -C

16. Let $2 - 3\sqrt{5} = x$, where x is a rational number ½ m

$\therefore\ 2 - x = 3\sqrt{5}$ or $\frac{2-x}{3} = \sqrt{5}$ ....(i) ½ m

As x is a rational number, so is $\frac{2-x}{3}$ 1 m

$\therefore$ LHS of (i) is rational but RHS of (i) is irrational

$\therefore$ Our supposition that x is rational is wrong

$\Rightarrow\ 2 - 3\sqrt{5}$ is irrational 1 m

17. Let the fraction be $\frac{x}{y}$, $y \neq 0$

$\therefore$ According to the question $x + y = 2y - 3$ 1 m

or $x = y - 3$.....................................(i)

Also, $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1$...(ii) 1 m

From (i) and (ii), $x = 4, y = 7$

$\therefore$ Required fraction $= \frac{4}{7}$ 1 m

OR

$\frac{4}{x} + 3y = 8$ .... (i), $\frac{6}{x} - 4y = -5$ ....(ii)

(i) × 3 $\Rightarrow \frac{12}{x} + 9y = 24$ and (ii) × 2 $\Rightarrow \frac{12}{x} - 8y = -10$ 1½ m

Solving to get $y = 2$ and $x = 2$ 1½ m

18. $S_{10} = -150$ and $S_{20} = -(150+550) = -700$ ½ m

$\therefore\ -150 = 5(2a + 9d)$ and $-700 = 10(2a + 19d)$ 1m

$\Rightarrow\ 2a + 9d = -30$ and $2a + 19d = -70$

$\Rightarrow\ d = -4$ and $a = 3$ 1m

$\therefore$ A . P is $3, -1, -5, \ldots\ldots\ldots\ldots$ ½ m

19. $AB^2 = AC^2 + BC^2 \ldots\ldots\ldots\ldots(i)$ and $AD^2 = AC^2 + DC^2$

$$= AC^2 + \frac{BC^2}{4}\left(\because\ DC = \frac{1}{2}BC\right)$$ 1+½ m

or $4\,AD^2 = 4\,AC^2 + BC^2 \ldots\ldots\ldots\ldots\ldots(ii)$ ½ m

From (i) and (ii), $AB^2 = AC^2 + 4\,AD^2 - 4\,AC^2$

or $AB^2 = 4\,AD^2 - 3\,AC^2$ 1 m

20. The given expression can be written as

$$\frac{\frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{1}{\frac{\sin A}{\cos A}\left(1 - \frac{\sin A}{\cos A}\right)}$$ 1 m

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$ ½ m

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A(\sin A - \cos A)} = \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$ 1 m

$$= \tan A + \cot A + 1$$ ½ m

OR

$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$ ½ m

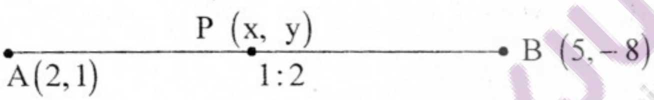
$$= \frac{1-\sin^2 A}{\sin A} \cdot \frac{1-\cos^2 A}{\cos A} = \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} = \sin A \cos A$$ 1½ m

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{1}{\dfrac{\sin^2 A}{\sin A \cos A} + \dfrac{\cos^2 A}{\sin A \cos A}} = \frac{1}{\tan A + \cot A} = \text{RHS}$$ 1 m

21. Correct construction of triangle Δ ABC 1 m

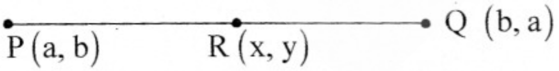
Correct construction of triangle similar to Δ ABC 2m

22. $\frac{AP}{AB} = \frac{1}{3} \Rightarrow \frac{AP}{PB} = \frac{1}{2}$ ½ m



$x = \frac{4+5}{3} = 3, \quad y = \frac{-8+2}{3} = -2 \Rightarrow P(3, -2)$ 1+½ =1½ m

P lies on $2x - y + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$ 1 m

23. 

If P, R, Q are collinear, ar (Δ PRQ) = 0 ½ m

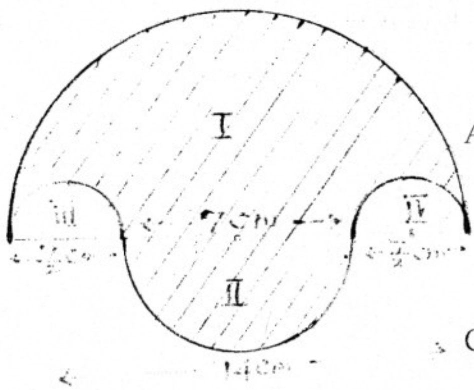
$\therefore \quad a(y-a) + x(a-b) + b(b-y) = 0$ 1 m

or $\quad ay - a^2 + ax - bx + b^2 - by = 0$

or, $(a-b)(x+y) = a^2 - b^2$ 1 m

$\Rightarrow \quad x + y = a + b$ ½ m

24.



Area of semi-circle I $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \text{cm}$

$= 77 \text{ cm}^2$ ½ m

Area of semi-circle II $= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{77}{4} \text{ cm}^2$ ½ m

Combined area of semi-circles III and IV

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \text{ cm}^2 = \frac{77}{8} \text{ cm}^2$ 1m

$\therefore$ Area of shaded region $= 77\left[\frac{1}{4} - \frac{1}{8} + 1\right] \text{ cm}^2$

$= \frac{693}{8} \text{ cm}^2$ or 86.625 cm^2 1m

OR

$AB^2 = AC^2 + BC^2$ (as $\angle ACB = 90°$)

$= 24^2 + 10^2 = 26^2 \Rightarrow AB = 26\text{cm}$ ½ m

$\therefore$ Area of semi-circle ACBOA $= \left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) \text{cm}^2$ 1m

$\therefore$ Area of $\Delta ACB = \left(\frac{1}{2} \times 24 \times 10\right) \text{cm} = 120 \text{ cm}^2$ ½ m

$\therefore$ Area of shaded region $= \left[\left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) - 120\right] \text{ cm}^2$

$= (265.33 - 120)$ or 145.33 cm^2 1m

25. Total number of cards = 18 ½ m

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13 – Five in number ½ m

$\therefore$ P (a prime no. less than 15) $= \frac{5}{18}$ 1m

(ii) numbers divisible by 3 and 5 is only 15 (one in number) ½ m

$\therefore$ P (a no. divisible by 3 and 5) $= \frac{1}{18}$ ½ m

SECTION D

26. Let the three consecutive numbers be $x, x+1, x+2$ 1 m

According to the question

$$x^2 + (x+1)(x+2) = 46$$

or $2x^2 + 3x - 44 = 0 \quad \Rightarrow \quad 2x^2 + 11x - 8x - 44 = 0$ 2 m

$\Rightarrow \quad (x-4)(2x+11) = 0$ 1 m

As x is positive $\Rightarrow \quad x = 4 \left(x = \frac{-11}{2} \text{ rejected} \right)$ 1 m

$\therefore$ The numbers are 4, 5, 6 1 m

OR

Let the two numbers be x, y where $x > y$

$\therefore \quad x^2 - y^2 = 88$ ....(i)

Also, $x = 2y - 5$ ....(ii)

} 2 m

From (i) and (ii), $(2y-5)^2 - y^2 = 88$

$\Rightarrow 3y^2 - 20y - 63 = 0$ 1 m

or $3y^2 - 27y + 7y - 63 = 0 \quad \Rightarrow (3y + 7)(y - 9) = 0$

$\Rightarrow$ y = 9, – 7/3 1 m

$x = 2y - 5 = 13$ or $x = -\frac{29}{3}$ 1 m

$\therefore$ The numbers are 13, 9 (Rejecting $x = \frac{-29}{3}$

and $y = -7/3$) 1 m

27. Correctly stated Given, To Prove, Construction and correct Figure $\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$ 2 m

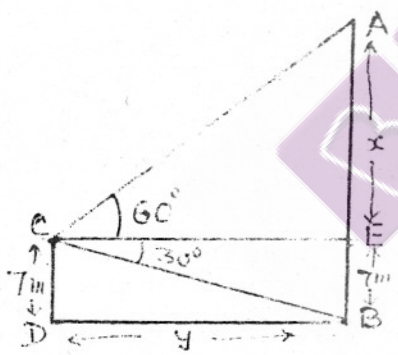
Correct Proof 2 m

Let ABC and PQR be two similar triangles ½ m

$$\frac{ar\,(ABC)}{ar\,(PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1 \Rightarrow \begin{array}{l} AB = PQ, \\ BC = QR,\ AC = PR \end{array}$$ 1½ m

$\therefore\ \Delta\, ABC \cong \Delta\, PQR$ (by SSS) ½ m

28. Let CD be the building and AB, the tower



Correct Figure 1m

Writing trigonometric equations

(i) $\frac{7}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow y = 7\sqrt{3}$

$= 12.124$ m 1½ m

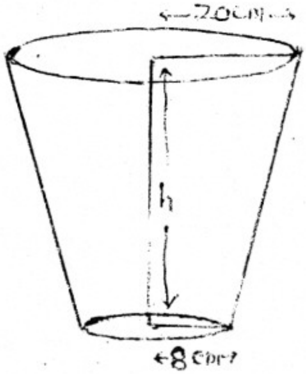
(ii) $\frac{x}{y} = \tan 60^\circ = \sqrt{3}$ 1½ m

or $\frac{x}{7\sqrt{3}} = \sqrt{3} \Rightarrow x = 21$ 1 m

$\therefore$ Height of tower = (21 + 7) m or 28 m 1 m

29. Volume of bucket = $10459\frac{3}{7}$ cm^3

$= \frac{73216}{7}$ cm^3



$$\therefore \quad \frac{73216}{7} = \frac{1}{3}\times\frac{22}{7}\times h\left[20^2+8^2+20\times 8\right] \quad 1m$$

$$\therefore\ h = \frac{73216}{7}\times\frac{21}{22}\times\frac{1}{624} = 16\text{ cm} \quad 1m$$

$$\ell^2 = h^2+\left(r_2-r_1\right)^2 = 16^2+\left(20-8\right)^2$$

$$= 400$$

$$\Rightarrow \ell = 20\text{ cm} \quad 1m$$

Total surface area of metal sheet used

$$= \frac{22}{7}\times 20\times\left(20+8\right)+\frac{22}{7}\left(8\right)^2\text{ cm}^2$$

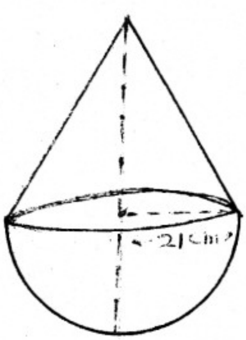
$$= \left(1760+\frac{1408}{7}\right)\text{ cm}^2$$

2 m

$$\therefore \text{ Cost of metal sheet } = \text{Rs}\left(1760+\frac{1408}{7}\right)\frac{14}{10}$$

$$= \text{Rs } 2745.60 \quad 1\text{ m}$$

OR



$$\text{Volume of hemisphere} = \frac{2}{3}\times\frac{22}{7}\times(21)^3\text{ cm}^3 \quad 1\text{ m}$$

$$\therefore \text{ Volume of cone} = \left(\frac{2}{3}\cdot\frac{2}{3}\ \frac{22}{7}\times 21\times 21\times 21\right)\text{ cm}^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times\left(21\right)^2\times h \quad 1\tfrac{1}{2}\text{ m}$$

$$\Rightarrow h = \frac{2\times 2\times \cancel{22}\times 7\times \cancel{21}}{\cancel{22}\times\cancel{21}} = 28\text{ cm} \quad 1\text{ m}$$

$$\therefore\ \ell^2 = h^2+r^2 = 28^2+21^2 = 1225 = \left(35\right)^2$$

$$\Rightarrow \ell = 35\text{ cm} \quad 1\text{ m}$$

Surface area of toy = $2\pi r^2 + \pi r \ell$

$$= \left(2 \times \frac{22}{7} \times 21 \times 21 + \frac{22}{7} \times 21 \times 35\right) \text{ cm}^2$$ 1½ m

$$= 5082 \text{ cm}^2$$

30.

Classes	0–10	10–20	20–30	30–40	40–50	50–60	60–70
class marks (x_i)	5	15	25	35	45	55	65
$d_i = \frac{x_i - 35}{10}$	–3	–2	–1	0	1	2	3
f_i	4	4	7	10	12	8	5
$f_i d_i$	–12	–8	–7	0	12	16	15

$\sum f_i = 50, \ \sum f_i d_i = 16$ 1 m

(i) $\bar{x} = 35 + \frac{16}{50} \times 10 = 38.2$ 1 m

(ii) Modal Class = 40 – 50 ½ m

Mode = $40 + \frac{12-10}{24-18} \times 10 = 43.33$ 1½ m

(iii) Median Class = 30 – 40 ½ m

Median = $30 + \frac{\frac{50}{2} - 15}{10} \times 10 = 40.00$ 1½ m

Note : If a candidate finds any two of the measures of central tendency correctly and finds the third correctly using Empirical formula, give full credit