

# CLASS – 10

## CHAPTER -5 Arithmetic Progression

### Introduction to AP

#### Sequences, Series and Progressions

- A **sequence** is a finite or infinite list of numbers following a certain pattern. For example: 1, 2, 3, 4, 5... is the sequence, which is infinite sequence of natural numbers.
- A **series** is the sum of the elements in the corresponding sequence. For example:  $1+2+3+4+5\dots$  is the series of natural numbers. Each number in a sequence or a series is called a term.
- A **progression** is a sequence in which the general term can be expressed using a mathematical formula.

#### Arithmetic Progression

An arithmetic progression (A.P) is a progression in which the **difference** between two **consecutive** terms is constant.

Example: 2, 5, 8, 11, 14.... is an arithmetic progression.

#### Common Difference

The difference between two consecutive terms in an AP, (*which is constant*) is the "**common difference**" (**d**) of an A.P. In the progression: 2, 5, 8, 11, 14 ...the common difference is 3.

As it is the difference between any two consecutive terms, for any A.P, if the common difference is:

- **positive**, the AP is **increasing**.
- **zero**, the AP is **constant**.
- **negative**, the A.P is **decreasing**.

#### Finite and Infinite AP

- A finite AP is an A.P in which the number of terms is finite. For example: the A.P: 2, 5, 8.....32, 35, 38

- An **infinite** A.P is an A.P in which the **number of terms is infinite**. For example: 2, 5, 8, 11.....

A finite A.P will have the last term, whereas an infinite A.P won't.

## General Term of AP

### The nth term of an AP

The nth term of an A.P is given by  $T_n = a + (n-1)d$ , where **a** is the first term, **d** is a common difference and **n** is the number of terms.

### The general form of an AP

The general form of an A.P is: **(a, a+d, a+2d, a+3d.....)** where **a** is the first term and **d** is a common difference. Here,  $d=0$ , OR  $d>0$ , OR  $d<0$

## Sum of Terms in an AP

### The formula for the sum to n terms of an AP

The sum to n terms of an A.P is given by:

$$S_n = n/2(2a + (n-1)d)$$

Where **a** is the first term, **d** is the common difference and **n** is the number of terms.

The sum of n terms of an A.P is also given by

$$S_n = n/2(a+l)$$

Where **a** is the first term, **l** is the last term of the A.P. and **n** is the number of terms.

## Arithmetic Mean (A.M)

The Arithmetic Mean is the simple average of a given set of numbers. The arithmetic mean of a set of numbers is given by:

$$A.M = \text{Sum of terms} / \text{Number of terms}$$

The arithmetic mean is defined for any set of numbers. The numbers need not necessarily be in an A.P.

## Basic Adding Patterns in an AP

The sum of two terms that are equidistant from either end of an AP is constant.

For example: in an A.P: 2,5,8,11,14,17...

$$T_1+T_6=2+17=19$$

$$T_2+T_5=5+14=19 \text{ and so on....}$$

Algebraically, this can be represented as

$$T_r+T_{(n-r)+1}=\text{constant}$$

## Sum of first n natural numbers

The **sum** of first **n** natural numbers is given by:

$$S_n=n(n+1)/2$$

This formula is derived by treating the sequence of natural numbers as an A.P where the first term (a) = 1 and the common difference (d) = 1.