CLASS – 10

CHAPTER -4 Quadratic Equations

Introduction to Quadratic Equations

Quadratic Polynomial

A polynomial of the form ax^2+bx+c, where a,b and c are real numbers and $a\neq0$ is called a quadratic polynomial.

Quadratic Equation

When we equate a quadratic polynomial to a constant, we get a quadratic equation.

Any equation of the form p(x)=c, where p(x) is a polynomial of degree 2 and c is a constant, is a quadratic equation.

The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2+bx+c=0$, where a,b and c are real numbers and $a\neq0$.
'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

Solving QE by Factorisation

Roots of a Quadratic equation

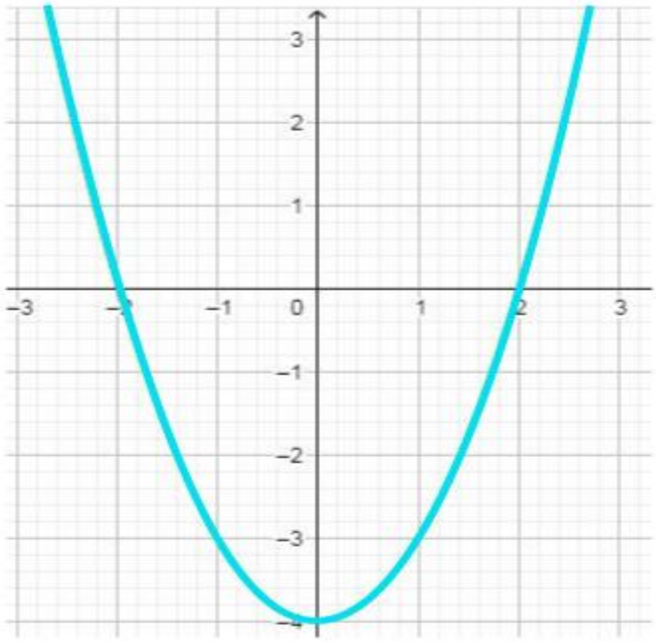
The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.

If α is a root of the quadratic equation $ax^2+bx+c=0$, then $a\alpha^2+b\alpha+c=0$.

A quadratic equation can have two distinct real roots, two equal roots or real roots may not exist.

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis.

Consider the graph of a quadratic equation $x^2-4=0$:



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation $x^2-4=0$
Note:

- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.
- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

Solving a Quadratic Equation by Factorization method

Consider a quadratic equation 2x2−5x+3=0
⇒2x2−2x−3x+3=0
This step is splitting the middle term
We split the middle term by finding two numbers (-2 and -3) such that their sum is equal to the coefficient of x and their product is equal to the product of the coefficient of x2 and the constant.
(-2) + (-3) = (-5)
And (-2) × (-3) = 6
2x2−2x−3x+3=0
2x(x−1)−3(x−1)=0
(x−1)(2x−3)=0

In this step, we have expressed the quadratic polynomial as a product of its factors.
Thus, $x = 1$ and $x = 3/2$ are the roots of the given quadratic equation.
This method of solving a quadratic equation is called the factorisation method.

Solving QE by Completing the Square

Solving a Quadratic Equation by Completion of squares method

In the method of completing the squares, the quadratic equation is expressed in the form $(x\pm k)^2=p^2$.

Consider the quadratic equation $2x^2-8x=10$
(i) Express the quadratic equation in standard form.
$2x^2-8x-10=0$

(ii) Divide the equation by the coefficient of x^2 to make the coefficient of x^2 equal to 1.
$x^2-4x-5=0$

(iii) Add the square of half of the coefficient of x to both sides of the equation to get an expression of the form $x^2\pm 2kx+k^2$.
$(x^2-4x+4)-5=0+4$

(iv) Isolate the above expression, $(x\pm k)^2$ on the LHS to obtain an equation of the form $(x\pm k)^2=p^2$
$(x-2)^2=9$

(v) Take the positive and negative square roots.
$x-2=\pm 3$

$x=-1$ or $x=5$

Solving QE Using Quadratic Formula

Quadratic Formula

Quadratic Formula is used to directly obtain the roots of a quadratic equation from the standard form of the equation.

For the quadratic equation $ax^2+bx+c=0$,

$x= [-b\pm \sqrt{(b^2-4ac)}]/2a$

By substituting the values of a,b and c, we can directly get the roots of the equation.

Discriminant

For a quadratic equation of the form $ax^2+bx+c=0$, the expression b^2-4ac is called the **discriminant**, (denoted by **D**), of the quadratic equation.
The **discriminant** determines the **nature of roots** of the quadratic equation based on the **coefficients** of the quadratic equation.

Solving using Quadratic Formula when D>0

Solve $2x^2-7x+3=0$ using the quadratic formula.
(i) Identify the coefficients of the quadratic equation. a = 2,b = -7,c = 3
(ii) Calculate the discriminant, b2−4ac
$D=(-7)^2-4\times2\times3= 49-24 = 25$
D> 0, therefore, the roots are distinct.
(iii) Substitute the coefficients in the quadratic formula to find the roots

$x= [-(-7)\pm \sqrt{((-7)^2-4(2)(3))}]/2(2)$

$x=(7 \pm5)/4$

x=3 and x= 1/2 are the roots.

Nature of Roots

Based on the value of the discriminant, $D=b^2-4ac$, the roots of a quadratic equation can be of three types.

Case 1: If **D>0**, the equation has two **distinct real roots**.

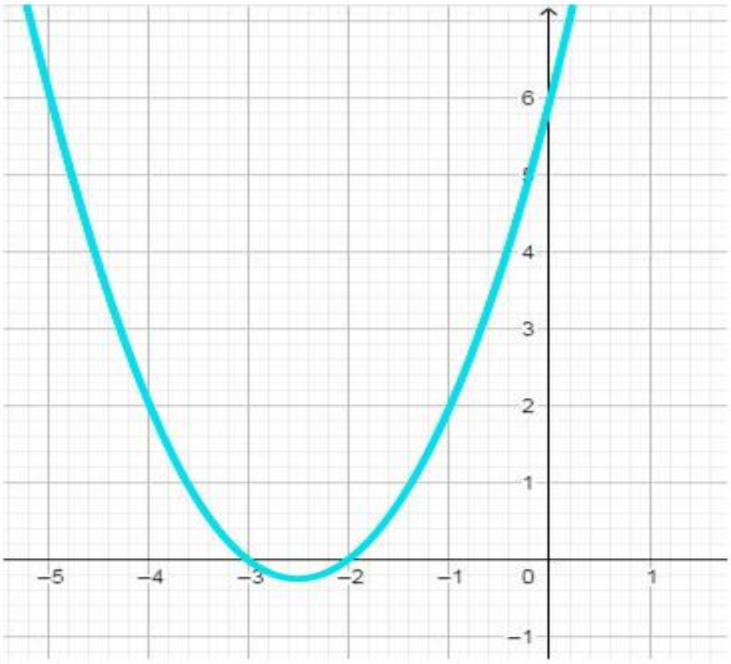
Case 2: If **D=0**, the equation has two **equal real roots**.

Case 3: If **D<0**, the equation has **no real roots**.

Graphical Representation of a Quadratic Equation

The graph of a quadratic polynomial is a parabola. The roots of a quadratic equation are the points where the parabola cuts the x-axis i.e. the points where the value of the quadratic polynomial is zero.

Now, the graph of $x^2+5x+6=0$ is:

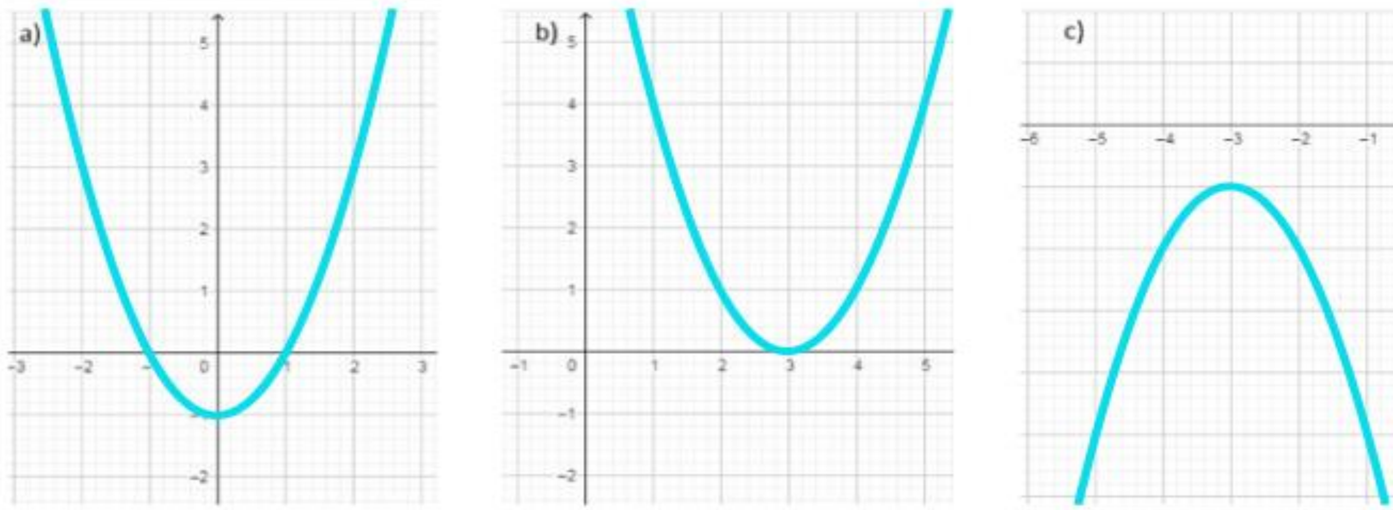


In the above figure, -2 and -3 are the roots of the quadratic equation $x^2+5x+6=0$.

For a quadratic polynomial ax^2+bx+c,

If **a>0**, the parabola opens **upwards**.
If **a<0**, the parabola opens **downwards.**
If **a = 0**, the polynomial will become a first-degree polynomial and its graph is linear.

The discriminant, $D=b^2-4ac$



Nature of graph for different values of D.

If **D>0**, the parabola cuts the x-axis at exactly two distinct points. The roots are distinct. This case is shown in the above figure in a, where the quadratic polynomial cuts the x-axis at **two distinct points.**

If **D=0**, the parabola just touches the x-axis at one point and the rest of the parabola lies above or below the x-axis. In this case, the roots are equal.
This case is shown in the above figure in b, where the quadratic polynomial touches the x-axis at **only one point**.

If **D<0**, the parabola lies entirely above or below the x-axis and there is no point of contact with the x-axis. In this case, there are no real roots.
This case is shown in the above figure in c, where the quadratic polynomial **neither cuts nor touches** the x-axis.

Formation of a quadratic equation from its roots

To find out the standard form of a quadratic equation when the roots are given:
Let α and β be the roots of the quadratic equation $ax^2+bx+c=0$. Then,
$(x-\alpha)(x-\beta)=0$
On expanding, we get,
$x^2-(\alpha+\beta)x+\alpha\beta=0$, which is the standard form of the quadratic equation.
Here, $a=1, b=-(\alpha+\beta)$ and $c=\alpha\beta$.

Sum and Product of Roots of a Quadratic equation

Let α and β be the roots of the quadratic equation $ax^2+bx+c=0$. Then,
Sum of roots $=\alpha+\beta=-b/a$
Product of roots $=\alpha\beta= c/a$