

CLASS – 10

CHAPTER -12 Areas related to Circles

Introduction

Area of a Circle

Area of a circle is πr^2 , where $\pi=22/7$ or ≈ 3.14 (can be used interchangeably for problem-solving purposes) and r is the radius of the circle.

π is the ratio of the circumference of a circle to its diameter.

Circumference of a Circle

The perimeter of a circle is the distance covered by going around its boundary once. The perimeter of a circle has a special name: Circumference, which is π times the diameter which is given by the formula $2\pi r$

Segment of a Circle

A circular segment is a region of a circle which is “cut off” from the rest of the circle by a secant or a chord.

Sector of a Circle

A circle sector/ sector of a circle is defined as the region of a circle enclosed by an arc and two radii. The smaller area is called the minor sector and the larger area is called the major sector.

Angle of a Sector

The angle of a sector is that angle which is enclosed between the two radii of the sector.

Length of an arc of a sector

The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

$$L = (\theta/360^\circ) \times 2\pi r$$

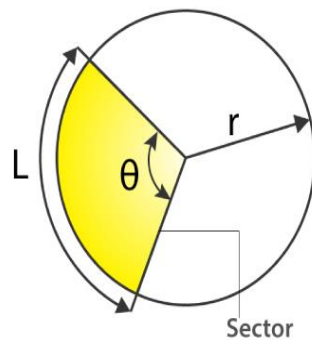
Where θ is the angle of sector and r is the radius of the circle.

Area of a Sector of a Circle

Area of a sector is given by

$$(\theta/360^\circ) \times \pi r^2$$

where $\angle \theta$ is the angle of this sector (minor sector in the following case) and r is its radius



Area of a sector

Area of a Triangle

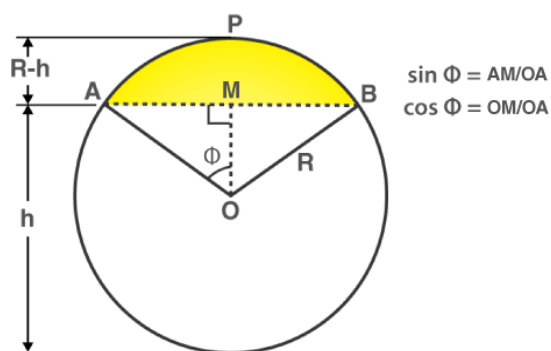
The Area of a triangle is,

$$\text{Area} = (1/2) \times \text{base} \times \text{height}$$

If the triangle is an equilateral then

$$\text{Area} = (\sqrt{3}/4) \times a^2 \text{ where "a" is the side length of the triangle.}$$

Area of a Segment of a Circle



Area of segment APB (highlighted in yellow)

$$= (\text{Area of sector OAPB}) - (\text{Area of triangle AOB})$$

$$= [(\phi/360^\circ) \times \pi r^2] - [(1/2) \times AB \times OM]$$

[To find the area of triangle AOB, use trigonometric ratios to find OM (height) and AB (base)]

Also, Area of segment APB can be calculated directly if the angle of the sector is known using the following formula.

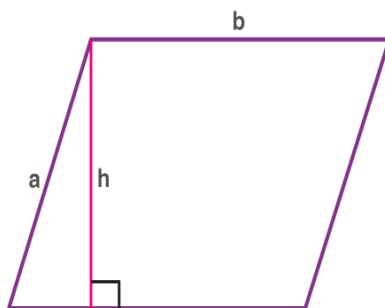
$$= \left[\frac{\theta}{360^\circ} \times \pi r^2 \right] - \left[r^2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2} \right]$$

Where θ is the angle of the sector and r is the radius of the circle

Visualizations

Areas of different plane figures

- Area of a square (side l) = l^2
- Area of a rectangle = $l \times b$, where l and b are the length and breadth of the rectangle
- Area of a parallelogram = $b \times h$, where “ b ” is the base and “ h ” is the perpendicular height.



parallelogram

Area of a trapezium = $\frac{(a+b) \times h}{2}$,

where

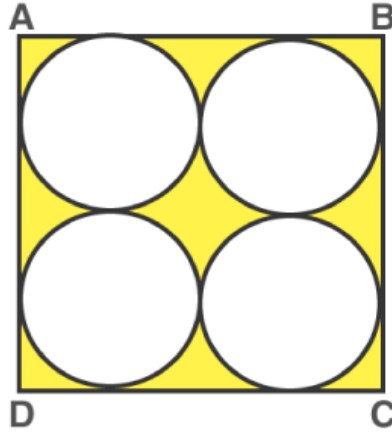
a & b are the length of the parallel sides

h is the trapezium height

Area of a rhombus = $\frac{pq}{2}$, where p & q are the diagonals.

Areas of Combination of Plane figures

For example: Find the area of the shaded part in the following figure: Given the ABCD is a square of side 28 cm and has four equal circles enclosed within.



Area of the shaded region

Looking at the figure we can visualize that the required shaded area
= A(square ABCD) - 4 × A(Circle).

Also, the diameter of each circle is 14 cm.

$$= (l^2) - 4 \times (\pi r^2)$$

$$= (28^2) - [4 \times (\pi \times 49)]$$

$$= 784 - [4 \times \frac{22}{7} \times 49]$$

$$= 784 - 616$$

$$= 168 \text{ cm}^2$$